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## Development of Cognitive Diagnostic Testing on Basic Arithmetic Operation

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### Abstract

A cognitive diagnostic test is an instrument for teachers to implement in measuring and evaluating during instruction in order to diagnose mastery learning for each attribute set for the cognitive model of individual students. The results of the diagnosis can be presented with a high degree of validity, convenience and speed. Hence, the present study was aimed at forming the cognitive diagnostic test on basic arithmetic operations, beginning by setting the desired attributes for measurement in the cognitive model, namely, the following four topics: addition, subtraction, multiplication and division of counted amounts and forming questions according to the set cognitive model. Next, the mental dimension is content validity by relying upon the discretion of experts.

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### 1. Introduction

According to the National Education Act, education should be focused on student-centred learning. Thus, it is necessary for teachers to be familiar with the background, personality, habits, health and families of students. In addition, academic background (existing knowledge, legibility, cognitive ability, skills, preference and interests) is also important for education planning. Thus, if a teacher knows a student's weaknesses and strengths, that a student has misconceptions and either does or does not understand particular subject matter, the teacher can correct the weaknesses and promote the strengths to thereby enable the student to develop to full capacity for successful

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learning with joy in learning and a desire to pursue further studies. In today's world, education administrators give importance to cognitive diagnostic testing. The term "cognitive" indicates an "attribute" composed of knowledge and skills used by students to solve test questions on subject content. The attribute is content-specific and arranged in hierarchy. Thus, cognitive diagnostic testing is designed according to a learning hierarchy used by students in problem-solving tests. The learning hierarchy is classified into attributes in a set cognitive model. Test scores tell whether or not the students possess mastery and, if so, in what attribute. This manner of test design offers a clearer picture. Mathematics is essential to human cognitive development. All students begin studying mathematics from numbers and basic arithmetic operations comprising addition, subtraction, multiplication and division of natural numbers. Each student possesses different ability to learn. Teachers need to know each student's ability to learn and, therefore, diagnose student flaws. The aforementioned issues sparked the researcher's interest in developing a cognitive model and cognitive diagnostic testing on basic arithmetic operations.

## 2. Literature review

The design of the cognitive diagnostic test began with the setting of a cognitive model composed of attributes used in the learning of basic arithmetic. Next, the cognitive model was created as designed and verified for content validity by experts. The documents and researches associated with the setting of cognitive model are presented as follows:

### 2.1 Cognitive Model Definition

Cognitive model means an attribute structure of knowledge and skills used by students to solve test problems correctly. Such attribute structures are arranged in hierarchy from basic to higher levels of attributes. Thus, the establishment of a cognitive model gives a clear structure linking cognitive attributes specifically for the content of each subject. It is a display of scores in detail according to the set cognitive attribute hierarchy (Leighton and Gierl, 2007; Gierl, Alves and Majeau, 2010; Pellegrino, Chudowsky, and Glaser, 2001, cited in Ketterlin – Geller and Yovanoff, 2009)

### 2.2 Method for Establishing a Cognitive Model

Cognitive model can be developed from 1) literature review or expert analysis; 2) using data that shows answers or methods for students; or 3) a combination of the first two methods. Each method differs according to the intention of test developer, time and resources used in establishing the cognitive model (Gotzmann, Roberts, Alves and Gierl, 2009; Gotzmann and Roberts, 2010; Cui and Cor, 2009 cited in Chu, Thompson, Bahry and Gotzmann, 2012).

## 3. Methodology

*Step 1* – Setting the cognitive diagnostic testing on basic arithmetic operations for addition, subtraction, multiplication and division of natural numbers in order to set the test's specific attributes as follows: 1) Reviewing documents and research related to basic arithmetic in four sub-subjects, namely, addition, subtraction, multiplication and division of natural numbers; use the data obtained to synthesize the initial cognitive model. 2) Submitting the cognitive model obtained by the document synthesis in No. 1 to content validity testing by five experts on 2.1) expert consensus on the cognitive model for addition, subtraction, multiplication and division of natural numbers; 2.2) preparation of a diagram on correlation among the attributes in each of the cognitive models and 2.3) additional opinions on each of the cognitive models. And 3) Preparing a diagram for the cognitive model for basic arithmetic operations.

*Step 2* – Creating cognitive diagnostic testing on basic arithmetic operations, preparing a test on the set cognitive model as follows: 1) The setting of Q-matrix is a display of the test under the conditions of the cognitive model set in Step 1. 2) The creation a test according to Q-matrix which involves multiple choice/fill-in-the-blank questions. One point is given if all spaces in one question are answered correctly, and 0 points are given if any space in a question is

incorrectly answered. And 3) Submitting the test obtained from the cognitive diagnostic test on basic arithmetic acquired to be examined for content validity by experts with consideration based on the congruence of questions and attributes to be measured (Item Objective Congruence: IOC).

#### 4. Results

##### *4.1 Results from the Setting of the Cognitive Model on Basic Arithmetic Operations*

The attribute synthesis obtained a total of four models. The correlation of each attribute in the cognitive model on basic arithmetic operations was set and content validity was verified by experts.

1) A cognitive model on addition of natural numbers composed of the following ten linear attributes: 1) Understanding the meaning of addition and using the “+” sign correctly; 2) Skills in adding two numbers with a sum not exceeding 9; 3) Skills in adding two numbers with a sum not exceeding 20 without carrying; 4) Skills in adding two numbers with a sum not exceeding 100 with 1 digit carrying; 5) Skills in adding two numbers with a sum not exceeding 100 with carrying; 6) Skills in adding two numbers with a sum not exceeding 1,000 without carrying; 7) Skills in adding two numbers with a sum not exceeding 1,000 with 1 digit carrying; 8) Skills in adding two numbers with a sum not exceeding 1,000 with 2 digits carrying; 9) Skills in adding two numbers with a sum not exceeding 100,000; and 10) Skills in adding two numbers with multiple digits.

2) A cognitive model on subtraction of natural numbers composed of the following 10 linear attributes: 1) Understanding the meaning of subtraction and using the “-” sign correctly; 2) Skills in subtracting two numbers with a minuend not exceeding 9; 3) Skills in subtracting two numbers with a minuend not exceeding 20 without distribution; 4) Skills in subtracting two numbers with a minuend not exceeding 100 without distribution; 5) Skills in subtracting two numbers with a minuend not exceeding 100 with distribution; 6) Skills in adding two numbers with a sum not exceeding 1,000 without distribution; 7) Skills in subtracting two numbers with a minuend not exceeding 1,000 with distribution; 8) Skills in subtracting two numbers with a minuend not exceeding 1,000 with distribution; 9) Skills in subtracting two numbers with a minuend not exceeding 100,000; and 10) Skills in subtracting two numbers with multiple digits.

3) A cognitive model on multiplication of natural numbers composed of the following eight linear attributes: 1) Understanding the meaning of multiplication and using the “x” sign correctly; 2) Skills in multiplying a one-digit number by a one-digit number; 3) Skills in multiplying a one-digit number by a two-digit number; 4) Skills in multiplying a one-digit number by a three-digit number; 5) Skills in multiplying a one-digit number by a four-digit number; 6) Skills in multiplying a one-digit number by a number with more than four digits; 7) Skills in multiplying a two-digit number by a two-digit number; 8) Skills in multiplying a number with more than one digit to a number with more than two digits.

4) A cognitive model on division of natural numbers composed of the following eight linear attributes: 1) Understanding the meaning of division and using the “÷” sign correctly; 2) Skills in dividing a dividend with no more than two digits by a divisor with one digit and the quotient is a one digit number (even division); 3) Skills in dividing a two-digit dividend by a divisor with one digit and the quotient is a two-digit number (even division); 4) Skills in dividing a two-digit dividend by a divisor with one digit and the quotient is a two-digit number (uneven division); 5) Skills in dividing a three-digit dividend by a one-digit divisor; 6) Skills in dividing a four-digit dividend by a one-digit divisor; 7) Skills in dividing with a two-digit divisor; 8) Skills in dividing with a three-digit divisor.

##### *4.2. Results from the creation of cognitive diagnostic testing on basic arithmetic operations.*

The creation of tests from the cognitive model on addition and subtraction of natural numbers can be divided in two levels, both of which have linear hierarchical structures. Level 1 was composed of Attributes 1-5 and Level 2 was composed of Attributes 6-10. For questions on cognitive diagnostic testing on addition and subtraction, Level 1 was composed of five sets of questions per subject as follows: Set 1 composed of questions to measure the Attribute 1. Set 2 composed of questions to measure the Attribute 1 and 2. Set 3 composed of questions to measure the Attribute 1, 2 and 3. Set 4 composed of questions to measure the Attribute 1, 2, 3 and 4. Set 5 composed of questions to measure

the Attribute 1, 2, 3, 4 and 5. And the questions of cognitive diagnostic test on addition and subtraction of natural numbers in Level 2 was composed of five sets of questions per subject as follows: Set 1 composed of questions to measure the Attribute 6. Set 2 composed of questions to measure the Attributes 6 and 7. Set 3 composed of questions to measure the Attributes 6, 7 and 8. Set 4 composed of questions to measure the Attributes 6, 7, 8 and 9. Set 5 composed of questions to measure the Attributes 6, 7, 8, 9 and 10. Test development from cognitive model on division and multiplication of natural numbers can be divided into two levels, each with linear hierarchical structure. Level 1 was composed of Attributes 1-4 and level 2 was composed of Attributes 5-8. The questions on the cognitive diagnostic test on multiplication and division of natural numbers in Level 1 were composed of 4 sets of questions per subject as follows: Set 1 composed of questions to measure the Attribute 1. Set 2 composed of questions to measure the Attribute 1 and 2. Set 3 composed of questions to measure the Attribute 1, 2 and 3. Set 4 composed of questions to measure the Attribute 1, 2, 3 and 4. The questions on the cognitive diagnostic test on multiplication and division of natural numbers in level 2 were composed of four sets of questions per subject as follows: Set 1 composed of questions to measure Attribute 5. Set 2 composed of questions to measure Attributes 5 and 6. Set 3 composed of questions to measure Attributes 5, 6 and 7. Set 4 composed of questions to measure Attributes 5, 6, 7 and 8.

## 5. Conclusion and discussion

In developing the cognitive diagnostic test on arithmetic operations, two sets of tests were acquired and diagnosed in the cognitive model at different levels. The tests are multiple choice/fill-in-the blank questions where 1 point was granted if all spaces in the questions were answered correctly and 0 points if a question was answered incorrectly in any space. Test 1 involved addition and subtraction of natural numbers, and it was used to diagnose Attributes 1 – 5 and multiplication and division of natural number were used to diagnose Attributes 1-4. Test 2 involved addition and subtraction of natural numbers and was used to diagnose Attributes 6-10 with multiplication and division of natural numbers used to diagnose Attributes 5-8. The method of test development results in the test was found to be capable of diagnosing student mastery correctly and clearly. The aforementioned finding concurs with the study of Gierl, Alves and Majeau (2010), who developed diagnostic testing on number and patterns and correlations for third and sixth grade students. The finding also concurs with the study of Gierl, Wang and Zhou (2008), who developed testing on algebraic operations for secondary level students from the setting of a cognitive model with content analysis by experts.

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